Roll No. _______ **Code No. Q-4393**

FOURTH SEMESTER EXAMINATION 2021-22

M.Sc. Mathematics

Paper - I

Integration Theory & Functional Analysis - II

Time : 3.00 Hrs. **Max. Marks : 80**

Total No. of Printed Page : 03 **Mini. Marks : 29**

Note: Question paper is divided into three sections. Attempt question of all three section as per direction. Distribution of Marks is given in each section.

Section - 'A'

Very short type question (in few words). **6x2=12**

Q.1 Attempt any six question from the following questions :

(i) Let N be a normed linear space and M a subspace of N then $\overline{M}$ is a .... of N.

(ii) The norm of identity operator on a normed linear space $N \neq \{0\}$ is ......

(iii) An operator T on Hilbert space H is normal if and only if .....

$\|Tx\| \quad Hx \in H$

(iv) If P is perpendicular projecton on M then $I-P$ is a .... on M^{+}.

(v) All norms are .... on a finite dimensional space.

(vi) State projection theorem.

(vii) State closed graph theorem.

(viii) Normed linear space is separable if it's .... is separable.

(ix) If $T_1, T_2 \in B(H)$ then $(T_1, T_2)* = ...$

(x) If S is non empty subset of a Hilbert space H Then

$S \cap S^1 \subset .....$

Section - 'B'

Short answer question (In 200 words) **4x5=20**

Q.2 Attempt any four question from the following questions :

(i) Prove that the linear space R^n of all $n-$tuples $x = (x_1, x_2, ...x_n)$ of Real numbers is normed linear space under the normed

$$\|x\| = \left(\sum_{i=1}^{n} |xi|^2 \right)^{1/2}$$

(ii) Let N and N' be normed linear space and let T be a linear transformation of N into N'. Then the inverse T^{-1} exists and is continuous of it's domain of definition if and only if $\exists\ k > 0$ such that

$k\|x\| \leq \|T(x)\|, \forall x \in N$

(iii) Prove that a linear operator on a finite dimensional normed space is continuous.

(iv) Give an example of a normed linear space which is not a Banach space.

(v) State and prove schwarz inequity.

(vi) An opertor on Hilbert space is normal it and only if it's Real and imaginary part commutes.

(vii) Prove that adjoint operator $T \to T*$ in $B(H)$ is Bijective.

Section - 'C'

Long answer/Essay type question. **4x12=48**

Q.3 Attempt any four question from the following questions :

(i) Let N be a non zero normed linear space and $s=\{x\in N:\|x\|\leq|\}$ be a linear subspace of N. Then prove the N is Banach space if and only if s is complete.

(ii) Let M be a linear subspace of a normed linear space N and let f be a functional defined on M then f can be extended to a functional f_0 to on whole space N such that :

$\|f\|=\|f_0\|$

(iii) Let $\{x_n\}$ be a weakly convergent sequenc in a normed space X. Then prove that :

(a) The weak limit of $\{x_n\}$ is unique.

(b) $\{\|x_n\|\}$ is a bounded sequence in R

(c) Every subsequence of $\{x_n\}$ converges weakly to the weak limit of $\{x_n\}$.

(iv) State and prove Riez representation theorem.

(v) If $\{e_i\}$ is an orthonormal set in Hilbert space H and if x is an arbitrary vector in H. Then prove that :

$x-\sum(x,ei)\ ei\perp ej$ for each j

(vi) Let T be an operator on a Hilbert space H then $\exists$ a unique operator on such that :

$(Tx,y)=(x,T^*y)\ \forall x,y\in H$

(vii) Prove that a closed convex subset C of a Hilbert space H contains a unique vector of smallest norm.

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